Dynamics of chiral primaries in $AdS_3 \times S^3 \times T^4$

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Abstract

We study in more detail the dynamics of chiral primaries of the
D1/D5 system . From the CFT given by the S_n orbifold a study of
correlators resulted in an interacting (collective) theory of chiral opera-
tors. In $AdS_3 \times S^3$ SUGRA we concentrate on general 1/2 BPS config-
urations described in terms of a fundamental string .We first establish
a correspondence with the linerized field fluctuations and then present
the nonlinear analysis. We evaluate in detail the symplectic form of
the general degrees of freedom in Sugra and confirm the appearance
of chiral bosons. We then discuss the apearance of interactions and
the cubic vertex,in correspondence with the S_N collective field theory
representation.

1 Introduction

Recently a dynamical descriptio of $\frac{1}{2}$ BPS configurations of $AdS_5 \times S^5$ SUGRA in terms of a fermionic (droplet) picture was acomplished[1, 2, 3]. The fermion dynamics is obtained from a reduction of $N = 4$ Yang-Mills theory on $R \times S^3$ in a manner similar to the holomorphic projection of the Hall effect. Its bosonic (collective droplet) representation is recovered completely by the ' bubbling' ansatz of Lin,Lunin and Maldacena [3] . The complete equivalence between two pictures is established through evaluation of the flux and the energy [3], [5, 6, 7, 8, 9, 10, 11, 12, 13] and also the symplectic form of the boundary degrees of freedom[4]. This dynamical map represents a most direct realization of the AdS/ CFT correspondence. It is expected to play a central role in further (nonperturbative) studies and provide a basis for further extensions [8]. The dynamics of $\frac{1}{2}$ BPS states of Supergravity on $AdS_3 \times S^3$ is similarly of substantial interest. The system represents $D1 - D5$ branes and is of relevance for microscopic study of black holes and the implementation of the AdS_3/CFT_2 correspondence. Early studies of the dynamics of chiral primaries were given in [17, 18] and in a series of papers [22, 23]. In [18] based on a study of three-point correlators on the S_n orbifold a cubic interaction hamiltonian was developed. In comparison with the single chiral collective boson of $AdS_5 \times S^5$ here the set of chiral primary fields is extended by a further(fermionic) structure associated with forms on T^4 or K^3 representing the compactification manifold. These fields summarize the collective effects of a simple 1d system which will be summarized and reviewed in sect.II. Recent studies on the Supergravity side have given further understanding of most general 1/2BPS configurations in $AdS_3 \times S^3$ gravity side one has the 'bubbling' ansatz describing $\frac{1}{2}$ BPS configurations. This ansatz is essentially the long string solution of [24, 25, 26].

In the present work we will discuss further the hamiltonian realization of the correspondence for the $D1 - D5$ system.We will study in some detail the dynamics of the $AdS_3 \times S^3$ 'bubbling' ansatz in Supergravity and perform a comparison with the collective theory of chiral bosons.For this we first present an evaluation of the symplectic form for the most general BPS configuration and establish the chiral boson description of these degrees of freedom from supergravity. In this we follow the method established by Maoz and Rychkov. We then to study the perturbative expansion of the hamiltonian and demonstrate at the quadratic and cubic level a correspondence with the collective boson description of [18].

2 Collective field theory of chiral primaries

We begin by giving a brief summary of the SCFT on symmetric product $S^N(X)$ and of the cubic(collective) representation for chiral primaries introduced in [16].the compactifiction manifold X is either T^4 or $K3$. The SCFT has (4,4) superconformal symmetry in both cases. We will work with T^4 for simplicity and most of the results extend simply to $K3$.

The field content of the theory consists of: $4N$ real free bosons $X_I^{a\dot{a}}$ representing the coordinates of the torus , their superpartners are $4N$ free fermions $\Psi_I^{\alpha\dot{a}}$, where $I = 1,..,N$, $\alpha, \dot{\alpha} = \pm$ are the spinorial S^3 indices, and $a, \dot{a} = 1, 2$ are the spinorial indices on T^4. Using the relation between the Fermi fields: $\Psi^{\alpha\dot{a}\,\dagger} = \Psi_{\alpha\dot{a}} = \epsilon_{\alpha\beta}\epsilon_{\dot{a}\dot{b}}\Psi^{\beta\dot{b}}$, the field content of the theory is determined to be $4N$ real free bosons and $2N$ Dirac free fermions, producing a total central charge $c = 6N$.

In the free CFT on T^4 one has non-trivial S_N invariant chiral primary operators which are constructed in correspondence with conjugacy classes of S_N . One has the basic twist operators for n free bosons: $X_I, I = 1..n$, defined through the OPE :

$$\partial X_I(z)\sigma_{(1..n)}(0) = z^{\frac{1}{n}-1}e^{-\frac{2\pi i}{n}I}\tau_{(1..n)}(0) + ..\tag{1}$$

They impose the boundary conditions :

$$X_I(z\,e^{2\pi i}, \bar{z}\,e^{-2\pi i}) = X_{I+1}(z, \bar{z}),\ I = 1..n-1,\ X_n(z\,e^{2\pi i}, \bar{z}\,e^{-2\pi i}) = X_1(z, \bar{z})\tag{2}$$

These twist operators play a distinguished role in the chiral ring, they can be used to generate the rest by using the ring structure. In the correspondence with gravity in $ADS_3 \times S^3$ they are in one-one correspondence with single particle states.

Let us begin with the operators appearing in the untwisted sector, $n = 1$:

$$\omega^{(0,0)} = 1,\tag{3}$$

$$\omega^a = \psi_0^{+a},\tag{4}$$

$$\omega^{\bar{a}} = \bar{\psi}_0^{+a},\tag{5}$$

$$\omega^{a,\bar{b}} = \psi_0^{+a}\bar{\psi}_0^{+b},\tag{6}$$

$$\omega^{aa',\bar{b}} = \psi_0^{+a}\psi_0^{+a'}\bar{\psi}_0^{+b},\tag{7}$$

$$\omega^{a,\bar{b}\bar{b}'} = \psi_0^{+a}\bar{\psi}_0^{+b}\bar{\psi}_0^{+b'},\tag{8}$$

$$\omega^{(2,2)} = \psi_0^{+1}\psi_0^{+2}\bar{\psi}_0^{+1}\bar{\psi}_0^{+2}.\tag{9}$$

The complet set of vertex operators can be written in terms of the 6 free scalar fields and the twist operators.One considers the cycle $(1..n)$ and

defines S_n invariant 6 dimensional vector observables (left and right) :

$$Y_L(z) = \frac{1}{n} \sum_{I=1..n} (X^1_{I\ L}, X^2_{I\ L}, X^3_{I\ L}, X^4_{I\ L}, \phi^1_I, \phi^2_I)(z), \qquad (10)$$

$$Y_R(\bar{z}) = \frac{1}{n} \sum_{I=1..n} (X^1_{I\ R}, X^2_{I\ R}, X^3_{I\ R}, X^4_{I\ R}, \bar{\phi}^1_I, \bar{\phi}^2_I)(\bar{z}), \qquad (11)$$

The simplest vertex operator consists of the twist operator for all 6 fields and having momenta along the 2 extra Φ dimensions:

$$O^{(0,0)}_{(1..n)}(z, \bar{z}) = e^{i(k_L Y_L(z) + k_R Y_R(\bar{z}))} \sigma_{(1..n)}(\Phi, X)(z, \bar{z}) \qquad (12)$$

where $k_L = (0,0,0,0,\frac{n-1}{2},\frac{n-1}{2})$, $k_R = (0,0,0,0,\frac{n-1}{2},\frac{n-1}{2})$ represents the left and right momenta in 6 dimensions. All the other chiral vertex operators are obtained by combining the states appearing in the untwisted sector with the above twisted operators. One has the following construction:first introduce the product

$$O^A_{(1..n)}(z, \bar{z}) \leftarrow O^A_{(1)}(z, \bar{z})\ O^{(0,0)}_{(1..n)}(z, \bar{z}) \qquad (13)$$

where A index takes care of the spinorial indices which already fully appear at untwisted level. It is useful to define a basis of forms in the target space X and spanning its $H^{(1,1)}(X)$; we will denote them as $\omega^r_{a\bar{a}}$ where r counts the forms (for example, $r = 1..4$ for T^4, and $r = 1..20$ for $K3$)) and $a, \bar{a} = 1, 2$ and they are X indices. Using this forms and summing over all permutations, it is possible to describe the scalar chiral primaries up to a normalization constant :

$$O^{(0,0)}_n(z, \bar{z}) = \frac{1}{(N!(N-n)!n)^{\frac{1}{2}}} \sum_{h \in S_N} O_{h(1..n)h^{-1}}(z, \bar{z}), \qquad (14)$$

$$O^r_n(z, \bar{z}) = \frac{1}{(N!(N-n)!n)^{\frac{1}{2}}} \sum_{h \in S_N} O^{a,\bar{a}}_{h(1..n)h^{-1}} \omega^r_{a\bar{a}}(z, \bar{z}), \qquad (15)$$

$$O^{(2,2)}_n(z, \bar{z}) = \frac{1}{4} \frac{1}{(N!(N-n)!n)^{\frac{1}{2}}} \sum_{h \in S_N} O^{ab,\bar{a}\bar{b}}_{h(1..n)h^{-1}} \epsilon_{ab} \epsilon_{\bar{a}\bar{b}}(z, \bar{z}), \qquad (16)$$

Of special use in construction of the hamiltonian are normal modes defined from the residues of the above operators : $a^0_n, a^r_n, a^{(2,2)}_n$,these obey canonical commutation relations

$$\left[a_n, a^\dagger_m \right] = \delta_{n,m} \qquad (17)$$

$$\left[a^r_n, a^{s\dagger}_m \right] = \delta_{n,m} \delta_{r,s} \qquad (18)$$

Their correlation functions were determined from the operator products in

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[16] ,they are given by

$$\langle a^{0\dagger}_{n+k-1} a^0_k a^0_n \rangle = \frac{1}{2} \frac{1}{\sqrt{N}} \left((n+k-1)\, nk \right)^{\frac{1}{2}} \qquad (19)$$

$$\langle a^{r\dagger}_{n+k-1} a^s_k a^0_n \rangle = \frac{1}{2} \frac{1}{\sqrt{N}} \left((n+k-1)\, nk \right)^{\frac{1}{2}} \qquad (20)$$

$$\langle a^{(2,2)\dagger}_{n+k-1} a^{(2,2)}_k a^0_n \rangle = \frac{1}{2} \frac{1}{\sqrt{N}} \left((n+k-1)\, nk \right)^{\frac{1}{2}} \qquad (21)$$

$$\langle a^{(2,2)\dagger}_{n+k-1} a^r_k a^s_n \rangle = -\frac{1}{2} \frac{1}{\sqrt{N}} \left((n+k-1)\, nk \right)^{\frac{1}{2}} \qquad (22)$$

$$\langle a^{(2,2)\dagger}_{n+k-1} a^0_k a^0_n \rangle = \frac{1}{2} \frac{1}{\sqrt{N}} \left((n+k-3)\, nk \right)^{\frac{1}{2}}. \qquad (23)$$

From the above correlation functions we read off the collective field Hamiltonian

$$H = \sum_{n>0} \left[n a^\dagger_n a_n + n a^{r\dagger}_n a^r_n + n a^{(2,2)\dagger}_n a^{(2,2)}_n \right]$$

$$+ \frac{1}{\sqrt{N}} \sum_{n,k>0} \sqrt{nk\,(n+k-1)} \left[a^\dagger_{n+k-1} a_k a_n + a^\dagger_k a^\dagger_n a_{n+k-1} \right]$$

$$+ \frac{1}{\sqrt{N}} \sum_{n,k>0} \sqrt{nk\,(n+k-1)} \left[a^{r\dagger}_{n+k-1} a^r_k a_n + a^{r\dagger}_k a^\dagger_n a^r_{n+k-1} \right]$$

$$+ \frac{1}{\sqrt{N}} \sum_{n,k>0} \sqrt{nk\,(n+k-1)} \left[a^{(2,2)\dagger}_{n+k-1} a^{(2,2)}_k a_n + a^{(2,2)\dagger}_k a^\dagger_n a^{(2,2)}_{n+k-1} \right]$$

$$+ \frac{1}{\sqrt{N}} \sum_{n,k>0} \sqrt{nk\,(n+k-1)} \left[a^{(2,2)\dagger}_{n+k-1} a^r_k a^r_n + a^{r\dagger}_k a^{r\dagger}_n a^{(2,2)}_{n+k-1} \right]$$

$$+ \frac{1}{\sqrt{N}} \sum_{n,k>0} \sqrt{nk\,(n+k-3)} \left[a^{(2,2)\dagger}_{n+k-3} a^\dagger_k a_n + a^\dagger_n a^{(2,2)}_{n+k-3} a_k \right]$$

The Hamiltonian takes the form

$$H_{coll} = \frac{1}{6} \sum_{n_1,n_2,n_3} \delta_{n_1+n_2+n_3,0} \alpha_{n_1} \alpha_{n_2} \alpha_{n_3} + \sum_n \alpha_n T_{-n}(S) + \cdots. \qquad (24)$$

We see that the (0,0) chiral primary field which is described by the modes $a^{(0,0)}_n$ is self interacting with a cubic interaction of a familiar collective fermion. The other fields which we denote as s^R_n, $R = 1 \ldots 5$ (related to the operators a^r, $a^{2,2}$ interact with α. This interacting theory is seen to be associated with the collective degrees of freedom consisting of N eigenvalue

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coordinates λ_i, $i = 1 \ldots N$ and the fermionic (zero modes) $\psi_i^{a,\alpha}$ with S_N symmetry. The collective fields given by

$$\Phi^{(p,q)}(x) = \sum_{i=1}^{N} \delta(x - \lambda_i) W^{(p,q)}(\psi_i, \bar{\psi}_i)$$

where $W^{(p,q)}$ are associated with the (p,q) forms on T^4 or K_3. In particular the form $(0,0)$, $W^{(0,0)} = 1$ leads to

$$\Phi(x) = \sum_{i=1}^{N} \delta(x - \lambda_i)$$

which is just the density of eigenvalues, we denote the other collective fields as S^R [1].

The structure of collective fields is then recognized as the one of the matrix-vector model. We have in general the density fields and the current which separate in terms of chiral components

$$\begin{aligned}
\Phi(x) &= y_+(x) - y_-(x) \\
J^\alpha(x) &= J_+^\alpha(x) - J_+^\alpha(x)
\end{aligned}$$

with the Hamiltonian

$$H = \int dx \left[\frac{1}{3}\left(y_+^3 - y_-^3\right) + x^2\left(y_+^3 - y_-^3\right) + y_+ J_+^2 + y_- J_-^2 + L' \right]$$

where L' stands for higher order (quartic) coupling . The commutation relations are those of a $U(1) \times U(1)$ chiral boson

$$\left[y_\pm(x), y_\pm(x') \right] = \pm 2\delta'(x - x')$$

and a Kac-Moody algebra. It represents a collective field theory of a matrix-vector type studied in [32].

It is simple to see how this general Hamiltonian encompases the interaction of chiral primaries written down above . First, the presence of the x^2 potential induces a classical background

$$\phi_0(x) = \frac{1}{\pi}\sqrt{\mu - x^2}$$

[1]At the orbifold point the $O(4)$ symmetry is manifestly visible while the $O(5)$ must have a nonlinear origin

which is interpreted as a one-point function in the CFT collective field theory (in later section we will reproduce this structure in SUGRA). After the shift

$$\phi(x) = \phi_0 + \sigma(x)$$

one obtains the cubic interaction

$$H_3 = \int d\tau \frac{1}{\phi_0^2} \sigma(x)^3$$

where τ is the time of flight coordinate. In terms of modes

$$H_3 = \sum_{n_1, n_2, n_3} f(n_1 + n_2 + n_3) \alpha(n_1) \alpha(n_2) \alpha(n_3)$$

with the form factor

$$f(n) = \frac{n}{\sinh \pi n}.$$

For the special case of observables (corresponding to chiral primaries) where one has momentum conservation $n = n_1 + n_2 + n_3 = 0$ we see

$$f(n) \to 1$$

resulting in a conserving, form factor . We emphasize that the collective field theory (and also gravity) have a more general set of generators. What we have found is a theory of a fermionic droplet interacting with current degrees of freedom living on the boundary of the droplet.

3   Linearized fields from SUGRA on $AdS_3 \times S^3$

In this section we will begin our discussion of the supergravity side by a usefull linearized analysis. At the linearized level the chiral primaries have been identified in [14]. The effective action of the degrees of freedom, that we are interested in, was given at the cubic level in [19, 20], the pp limit was studied in [21]. We will identify these chiral primaries by expanding the exact nonlinear solution of Sugra constructed in [27].

We consider the action of IIB supergravity in the string frame

$$S = \int dx^{10} \sqrt{-g} \left[e^{-2\Phi} \left(R + 4(\nabla\Phi)^2 \right) - \frac{1}{12} H^2 \right]$$

where H denotes the field strength of the RR two form $H = dC^{(2)}$. The D1-D5 system is described by the the profile of a curve $F_i(u)$, $u = 0 \ldots 2\pi Q_5$, $i = 1, \ldots, 4$.

The solution in the 10D string frame is given by

$$ds^2 = \frac{1}{\sqrt{f_1 f_5}} \left[- \left(dt - A_i dx^i \right)^2 + \left(dy + B_i dx^i \right)^2 \right] + \sqrt{f_1 f_5} d\mathbf{x}^2 + \sqrt{\frac{f_1}{f_5}} d\mathbf{z}^2$$

$$e^{2\Phi} = \frac{f_1}{f_5}$$

$$C_{ti}^{(2)} = \frac{B_i}{f_1}, \quad C_{ty}^{(2)} = \frac{1}{f_1} - 1$$

$$C_{iy}^{(2)} = -\frac{A_i}{f_1}, \quad C_{ii}^{(2)} = \mathcal{C}_{ij} - \frac{1}{f_1} \left(A_i B_j - A_j B_i \right)$$

where the harmonic functions are given by

$$f_5 = 1 + \frac{Q_5}{L} \int_0^L \frac{du}{|x - F|^2}$$

$$f_1 = 1 + \frac{Q_5}{L} \int_0^L \frac{|\dot{F}|^2 du}{|x - F|^2}$$

$$A_i = -\frac{Q_5}{L} \int_0^L du \frac{F_{i,u}}{|x - F|^2}$$

and the fields B_i and $\mathcal{C}_{ij}$ are obtained by solving

$$d\mathcal{C} = -*_4 \, df_5$$
$$dB = -*_4 \, dA$$

where the Hodge duality is meant for the the flat space spanned by x^i. In our calculations we have used the solution for B_i given by

$$B_1 = B_2 = 0$$
$$B_3 = -\frac{x_4}{x_3^2 + x_4^2} Z$$
$$B_4 = \frac{x_3}{x_3^2 + x_4^2} Z$$
$$Z = \frac{Q_5}{L} \int_0^L du \frac{(x_2 - F_2) \partial_u F_1 - (x_1 - F_1) \partial_u F_2}{|x - F|^2}.$$

The one brane charge is given by

$$Q_1 = \frac{Q_1}{L} \int_0^L |\dot{F}|^2 du, \text{ and}$$
$$L = 2\pi Q_5.$$

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The decoupling limit, which will give $AdS_3 \times S^3$ as the vacuum, is taken as

$$
\begin{aligned}
f_5 &= 1 + \frac{Q_5}{L} \int_0^L \frac{du}{|x - F|^2} \approx \frac{Q_5}{L} \int_0^L \frac{du}{|x - F|^2} \\
f_1 &= 1 + \frac{Q_5}{L} \int_0^L \frac{|\dot{F}|^2 \, du}{|x - F|^2} \approx \frac{Q_5}{L} \int_0^L \frac{|\dot{F}|^2 \, du}{|x - F|^2}.
\end{aligned}
$$

We will be interested in regular [27] configurations with $F_3 = F_4 = 0$. As it was described in [27] we expect to get the two chiral primaries coming from the combination of the metric and the selfdual part of the flux for the first one, which we will call σ, and the combination of the dilaton and the anti-selfdual part of the flux for the second one which we will call s^5.

For our purposes we have found useful the change of variables from $F_i(u)$ to $\rho(\phi)$, $u(\phi)$ according to

$$
\begin{aligned}
\rho(\phi) &= \sqrt{F_1^2 + F_2^2} \\
\phi &= \tan\left(\frac{F_2(u)}{F_2(u)}\right).
\end{aligned}
$$

The reader may find some details of the perturbative calculation in the appendix where we also repeat the treatment of the circular profile. In the next section we match space time field modes to fourier modes of the parametrization ρ, u.

In order to see the chiral primary s^5 we have to look at either the dilaton or the anti-self dual part of the RR field H. Choosing the first option we see that at the first nontrivial order

$$
\begin{aligned}
e^{2\Phi} &\approx \frac{f_1^0}{f_5^0} + \frac{1}{f_5^0}\delta f_1 - \frac{f_1^0}{f_5^{0^2}}\delta f_5 \\
&= \frac{Q_1}{Q_5}\left[1 + \frac{2\left(r^2 + \cos^2\theta\right)}{2\pi}\int_0^{2\pi} d\tilde{\phi}\frac{\rho_0^{-1}\delta\rho - \delta u_{,\tilde{\phi}}/Q_5}{d^2}\right]
\end{aligned}
$$

¿From the above we see that the combination that is relevant to the chiral primary s^5 is

$$
\delta b = \delta\rho - \frac{\rho_0}{Q_5}\delta u_{,\tilde{\phi}} = \sum_n b_n e^{in\phi}, \; b_{-n} = b_n^\dagger
$$

The above expansion gives us the fluctuation of the dilaton

$$
\delta\Phi = \sum_n b_n \left(\frac{\sin\theta}{\sqrt{r^2 + 1}}\right)^n e^{in\phi}.
$$

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We can read off the field s^5 from the known expression [14]

$$\delta\Phi\left(t,r,\theta,\phi\right) = 2\sum_n |n|\, s_n^5\left(t,r\right)\sin^n\theta e^{in\phi}$$

where we have only included only the relevant for us highest and lowest weight spherical harmonics in the sum.

In order to extract the chiral primary σ we should be looking at either the metric of the self dual part of the field strength of the RR field $C^{(2)}$. Choosing the first way we want to extract the peturbed metric on the sphere and bring it to the Lorentz- De Donder gauge. In this gauge the relevant degree of freedom just scales the volume of the sphere. Expanding the metric around the ground state and looking at the sphere components we have that

$$
\begin{aligned}
\frac{d\tilde{s}_{S^3}^2}{Q_1 Q_5} &= -\frac{\delta h}{2 h_0^{\frac{3}{2}}}\left(A_\phi^{0\,2} d\phi^2 + B_\psi^{0\,2} d\psi^2\right) + \frac{\delta h}{2 Q_1 Q_5 h_0^{\frac{1}{2}}} d\mathbf{x}^2 \\
&\quad - \frac{1}{\sqrt{h_0}}\left(2 A_\phi^0 \delta A_\phi d\phi^2 + 2 A_\phi^0 \delta A_\theta d\theta d\phi\right) + \frac{2 B_\psi^0}{\sqrt{h_0}}\delta B_\psi d\psi^2
\end{aligned}
$$

where we have set $h = f_1 f_5$. After putting everything together we have that

$$
\begin{aligned}
\delta g_{\theta\theta} &= \frac{r^2 + \cos^2\theta}{2\sqrt{h_0}}\delta h = \frac{r^4 - \cos^4\theta}{2\pi}\int_0^{2\pi} d\tilde{\phi}\frac{\delta\rho}{d^4} \\
&= \frac{r^2 - \cos^2\theta}{r^2 + \cos^2\theta}\sum_n a_n e^{in\phi}\left(\frac{\sin\theta}{\sqrt{r^2+1}}\right)^n\left[|n| + \frac{r^2 + 1 + \sin^2\theta}{r^2 + \cos^2\theta}\right] \\
\delta g_{\psi\psi} &= \cos^2\theta\left[\frac{r^2 - \cos^2\theta}{2\sqrt{h_0}}\delta h + 2\delta B_\psi\right] \\
&= \frac{r^2 + \sin^2\theta + 1}{2\pi}\int_0^{2\pi} d\tilde{\phi}\frac{\delta\rho}{d^4} - \frac{2\sqrt{r^2+1}\sin\theta}{2\pi}\int_0^{2\pi} d\tilde{\phi}\frac{\sin\left(\tilde{\phi}-\phi\right)}{d^2}\delta\rho_{,\tilde{\phi}} \\
&= \cos^2\theta\sum_n a_n e^{in\phi}\left(\frac{\sin\theta}{\sqrt{r^2+1}}\right)^n\left[|n| + \frac{r^2 + \sin^2\theta + 1}{r^2 + \cos^2\theta}\right].
\end{aligned}
$$

Some of the details of this calculation along with some notation convetions are explained in the appendix. After performing the gauge transformation

$$\theta \to \theta + \frac{\sin\theta\cos\theta}{r^2 + \cos^2\theta}\sum_n a_n e^{in\phi}\left(\frac{\sin\theta}{\sqrt{r^2+1}}\right)^n$$

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the metric components of the sphere are scaled as

$$
\begin{aligned}
\delta g_{mn}\left(t, r, \theta, \phi\right) &= \bar{g}_{mn}\left(\theta\right) \sum_n \left(|n| + 1\right) a_n e^{in\phi} \left(\frac{\sin\theta}{\sqrt{r^2 + 1}}\right)^n \\
&= 2\bar{g}_{mn}\left(\theta\right) \sum_n |n| \, \sigma_n\left(t, r\right) \sin^n \theta e^{in\phi}
\end{aligned}
$$

where $\bar{g}_{mn}$ is metric on S^3.

The previous considerations helped us translate degrees of freedom of the curve to spacetime fields of minimal six dimensional SUGRA coupled to one tensor multiplet. The correctly normalized action for the above chiral primaries is given by

$$
\begin{aligned}
S = \quad & \sum_n \frac{2\,|n|}{|n| + 1} \int_{AdS_3} dx^3 \left[\left(|n| + 1\right) \left(\left(\nabla s_n^5\right)^2 + |n| \left(|n| - 2\right) \left(s_n^5\right)^2\right)\right] \\
+ \quad & \sum_n \frac{2\,|n|}{|n| + 1} \int_{AdS_3} dx^3 \left[\left(|n| - 1\right) \left(\left(\nabla \sigma_n\right)^2 + |n| \left(|n| - 2\right) \left(\sigma_n\right)^2\right)\right]
\end{aligned}
$$

After restricting the system to the supersymmetric case we have the canonical expansion of the fields

$$
\begin{aligned}
s^5 &= \sum_n s_n^5 Y^n = \frac{1}{2} \sum_n \frac{1}{\sqrt{2\,|n|}} e^{in\phi} \left(\frac{\sin\theta}{\sqrt{r^2 + 1}}\right)^n c_n \\
\sigma &= \sum_n \sigma_n Y^n = \frac{1}{2} \sum_n \sqrt{\frac{1}{2\,|n|} \frac{|n| + 1}{|n| - 1}} e^{in\phi} \left(\frac{\sin\theta}{\sqrt{r^2 + 1}}\right)^n d_n.
\end{aligned}
$$

Comparing the components of the SUGRA fields that we previously calculated we see that the identification is

$$
b_n = \sqrt{\frac{|n|}{2}} c_n
$$

$$
a_n = \sqrt{\frac{1}{2} \frac{|n|}{n^2 - 1}} d_n
$$

The angular momentum of the full solution is given by

$$
\begin{aligned}
J &= J_L + J_R = \frac{1}{2\pi} \int_0^L du \left(F_1\left(u\right) \dot{F}_2\left(u\right) - \dot{F}_1\left(u\right) F_2\left(u\right)\right) - Q_1 Q_5 \\
&= \frac{1}{2\pi} \int_0^{2\pi} d\phi \rho^2\left(\phi\right) - Q_1 Q_5
\end{aligned}
$$

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At this point we would like to separate the zero mode dependence of $\rho(\phi)$ as

$$\rho(\phi) = \rho_0 + \delta\rho(\phi)$$
$$\int_0^{2\pi} d\phi \, \delta\rho(\phi) = 0.$$

The result for the angular momentum is

$$J = \rho_0^2 + \sum_{n>1} a_n a_{-n} - Q_1 Q_5$$

For the case of the $D1 - D5$ system we need to keep $Q_1 Q_5$ fixed, which gives the constrain

$$N = \frac{Q_5^2}{L} \int_0^L du |\dot{F}|^2 = \frac{Q_5}{2\pi} \int_0^{2\pi} d\phi \, \frac{\rho_{,\phi}^2 + \rho^2}{u_{,\phi}}$$

We can solve the above constrain for ρ_0 perturbatively in fluctuations of ρ and u.

$$\rho_0^2 \approx N + \frac{1}{2\pi} \int_0^{2\pi} d\phi \, \delta\rho\delta\rho_{,\phi\phi} - \frac{1}{2\pi} \int_0^{2\pi} d\phi \left(\delta\rho - \frac{\rho_0}{Q_5} \delta u_{,\tilde{\phi}} \right)^2$$

Using the expression that we previously found for the angular momentum we find

$$
\begin{aligned}
J &= -\frac{1}{2\pi} \int_0^{2\pi} d\phi \left(\delta\rho - \frac{\rho_0}{Q_5} \delta u_{,\tilde{\phi}} \right)^2 + \frac{1}{2\pi} \int_0^{2\pi} d\phi \left(\delta\rho\delta\rho_{,\phi\phi} + \delta\rho^2 \right) \\
&= -\sum_{n>0} b_n b_{-n} - \sum_{n>1} \left(n^2 - 1 \right) a_n a_{-n} \\
&= -\sum_{n>0} n c_n c_{-n} - \sum_{n>1} n d_n d_{-n}
\end{aligned}
$$

4 Non-linear analysis

The f1f5 solution provids a general non-singular 1/2 BPS solution of 6D Sugra .It is parametrized by the string-like coordinates F whose dynamics is of interest. One can think of this in analagy with extended (soliton-like) solutions of nonlinear field theory[46]. In this spirint one would like to establish the dynamics of general 1/2 BPS configuration in Supergravity by direct evaluation of the action.Some aspects of the result could be infered from the extended string-like (or superube) nature othe solution. The direct Sugra

confirmation is nevertheless of importance, especialy regarding the question of correlation and interactions in the theory. With this purpose we begin first by direct evaluation the symplectic form from Sugra. In this we will follow the methods of [4] which are based on the covariant ZCW symplectic form [33, 34]. According to this method which is generaly applicable to every field theory described by a Lagrangian density $L\left(\phi_i, \partial_l \phi_i\right)$, with fields and their first derivatives, the symplectic form is given by

$$J^l = \frac{\delta L\left(\phi_i, \partial_l \phi_i\right)}{\delta \partial_l \phi_i} \wedge \delta \phi_i.$$

As it was pointed out in [4]. while using the above expression one has to use a regular gauge choice for both the fields ϕ_i and the variations $\delta \phi_i$. The effect of gauge transformations is the addition of total derivative terms in the vector symplectic density. In the case where one has well behaved transformations these total derivative terms can be dropped. In the case where one wants to correct a singular gauge choice for the filed variations $\delta \phi_i$ these total derivative terms cannot be dropped [4].

For the space of solutions $F_3 = F_4 = 0$ that we are interested in we will use the dimensionally reduced 6d Einstein frame action

$$S = \int dx^6 \sqrt{-g} \left[R - \frac{1}{12} e^{\sqrt{2}a} H^2 - \frac{1}{2} \left(\nabla a\right)^2 \right] =$$
$$\int dx^6 \sqrt{-g} \left[g^{mn} \left(\Gamma_{ml}^a \Gamma_{an}^l - \Gamma_{mn}^a \Gamma_{la}^l \right) - \frac{1}{12} e^{\sqrt{2}a} H^2 - \frac{1}{2} \left(\nabla a\right)^2 \right]$$

in order to evaluate the symplectic form.

In the above action we have dropped the total derivative terms that come from partial integrations necessary to reach the first derivative form of the action. One could check that such a total derivative term [35] gives no contribution. The $D1 - D5$ solution in the ten dimensional string frame is given by

$$d\tilde{s}^2 = \frac{1}{\sqrt{f_1 f_5}} \left[-\left(dt - A_i dx^i \right)^2 + \left(dy + B_i dx^i \right)^2 \right] + \sqrt{f_1 f_5} d\mathbf{x}^2 + \sqrt{\frac{f_1}{f_5}} d\mathbf{z}^2$$
$$e^{2\Phi} = \frac{f_1}{f_5}$$
$$C_{ti}^{(2)} = \frac{B_i}{f_1}, \ C_{ty}^{(2)} = \frac{1}{f_1} - 1$$
$$C_{iy}^{(2)} = -\frac{A_i}{f_1}, \ C_{ij}^{(2)} = \mathcal{C}_{ij} - \frac{1}{f_1} \left(A_i B_j - A_j B_i \right).$$

The harmonic functions that appear in the above equations are

$$f_5 = 1 + \frac{Q_5}{L} \int_0^L \frac{du}{|x - F|^2}$$

$$f_1 = 1 + \frac{Q_5}{L} \int_0^L \frac{|\dot{F}|^2 du}{|x - F|^2}$$

$$A_i = -\frac{Q_5}{L} \int_0^L du \frac{F_{i,u}}{|x - F|^2}$$

and the fields B_i and $\mathcal{C}_{ij}$ are obtained by solving

$$d\mathcal{C} = - *_4 df_5$$
$$dB = - *_4 dA$$

where the Hodge duality is meant for the flat space spanned by x^i. In order to transform the solution to the ten dimensional Einstein frame we need to scale the metric as

$$g_{\mu\nu} = e^{\frac{\phi}{2}} \tilde{g}_{\mu\nu}.$$

It is true that the above transformation generates total derivative terms. The behavior of the dilaton near the curve $\mathbf{x} = \mathbf{F}(u)$ is such that possible contributions are zero for our case. Going back to the six dimensional Einstein frame we have the ansatz

$$ds^2 = -\frac{1}{\sqrt{f_1 f_5}} \left(dt - A_i dx^i\right)^2 + \left(dy + B_i dx^i\right)^2 + \sqrt{f_1 f_5} d\mathbf{x}^2$$
$$a = \sqrt{2}\phi$$

The scalar a comes from a combination of the ten dimensional dilaton with the volume of the torus. We will need the solution for B_i which is given by

$$B_1 = B_2 = 0$$
$$B_3 = -\frac{x_4}{x_3^2 + x_4^2} Z$$
$$B_4 = \frac{x_3}{x_3^2 + x_4^2} Z$$
$$Z = \frac{Q_5}{L} \int_0^L du \frac{(x_2 - F_2)\partial_u F_1 - (x_1 - F_1)\partial_u F_2}{|x - F|^2}$$

We need to observe that in this gauge choice

$$\nabla A = \nabla B = A \cdot B = 0.$$

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Fixing our Cauchy surface by $l = t$ we have

$$J^t = J_G^t + J_H^t + J_a^t + J_{bdry}^t$$

where

$$
\begin{aligned}
J_G^l &= -\delta \Gamma_{mn}^l \wedge \delta \left(\sqrt{-g} g^{mn} \right) + \delta \Gamma_{mn}^n \wedge \delta \left(\sqrt{-g} g^{lm} \right) \\
J_H^l &= -\frac{1}{2} \delta \left(\sqrt{-g} e^{\sqrt{2}a} H^{lmn} \right) \wedge \delta \left(C_{mn} \right) \\
J_a^l &= -\delta \left(\sqrt{-g} g^{lm} \partial_m a \right) \wedge \delta \left(a \right)
\end{aligned}
$$

and J_{bdry}^t is the contribution that comes from the gauge transformation which is necessary to show regularity of the metric δg_{mn} and the gauge field δC_{mn}. We will analyze this term separately. As we show in the appendix the bulk contribution is given by

$$J_{bulk}^t = -\int d^4 x \partial_d \left\{ \frac{1}{2} \epsilon_{ijad} \delta \left[\frac{A^a}{f_5} \right] \wedge \delta \left(C_{ij} \right) \right\}.$$

The above total integral receives contribution from a surface of $S^1 \times S^2$ topology which describes a "tube" surrounding the ring at $\mathbf{x} = \mathbf{F}(u)$. As we show in the appendix the value of this integral is given by

$$\int d^4 x \partial_d \left[\frac{1}{2} \epsilon_{ijad} \delta \left[\frac{A^a}{f_5} \right] \wedge \delta \left(C_{ij} \right) \right] = -2\pi \int_0^L du \frac{1}{\left| \dot{F} \right|^2} \left(\dot{F}_1 \delta \dot{F}_1 + \dot{F}_2 \delta \dot{F}_2 \right) \wedge \left(\dot{F}_1 \delta F_1 + \dot{F}_2 \delta F_2 \right).$$

4.1 Contribution coming from regularity gauge transformation

In order to show regularity of the fields under variation we need to perform gauge transformations for both the coordinates and the two form gauge field. For the present case we find that the only transformation that generates some finite contribution is the general coordinate transformation

$$x^i \rightarrow x^i + \xi^i \left(x^j, \delta F_j, \delta \dot{F}_j \right)$$

that we have to perform in order to show regularity of the metric $g_{mn} + \delta g_{mn}$. The above transformation generates total derivative term contribution in the symplectic form [33]. The contribution coming from infinity can be discarded since these terms would be proportional to the variation of the

charges that we keep fixed. The place that we need to specify the coordinate transformation is near the curve $x_i = F_i$. The transformation

$$\delta C_{mn} \quad \rightarrow \quad \delta C_{mn} + \xi^p H_{pmn} - 2\partial_{[m}\left(\xi^p C_{n]p}\right)$$
$$\delta g_{mn} \quad \rightarrow \quad \delta g_{mn} + \nabla_{(m}\xi_{n)}$$

close to the curve is given by

$$\xi^i = \delta F^i - \frac{\left(\vec{x} - \vec{F}\right)\cdot\delta\vec{l}}{|\dot{F}|^2}l^i - \frac{\left(\vec{x} - \vec{F}\right)\cdot\vec{l}}{|\dot{F}|^2}\delta l^i + 2\frac{\left(\vec{x} - \vec{F}\right)\cdot\vec{l}}{|\dot{F}|^4}\left(\vec{l}\cdot\delta\vec{l}\right)l^i$$
$$-\frac{\left(\vec{x} - \vec{F}\right)\cdot\delta\vec{n}}{|\dot{F}|^2}n^i - \frac{\left(\vec{x} - \vec{F}\right)\cdot\vec{n}}{|\dot{F}|^2}\delta n^i + 2\frac{\left(\vec{x} - \vec{F}\right)\cdot\vec{n}}{|\dot{F}|^4}\left(\vec{n}\cdot\delta\vec{n}\right)n^i$$
$$\vec{l}\left(u\right) = \left(\ \dot{F}_1\left(u\right),\quad \dot{F}_2\left(u\right),\quad 0,\quad 0\ \right)$$
$$\vec{n}\left(u\right) = \left(\ \dot{F}_2\left(u\right),\quad -\dot{F}_1\left(u\right),\quad 0,\quad 0\ \right)$$

The vector $\vec{l}$ is tangent to the curve $\vec{F}$ and the vector $\vec{n}$ is one of its normal vectors. A way to see this is, is to say that $g_{mn} + \delta g_{mn}$ is still in the family of solutions that we consider with $\mathbf{F} \rightarrow \mathbf{F} + \delta\mathbf{F}$. In order to show regularity we would have to follow steps similar to [27]. Starting from the background metric g_{mn} we would need to perform the above mentioned coordinate transformation in order to show that the total metric $g_{mn} + \delta g_{mn}$ is regular.

Unlike the case of the gauge field gauge transformations, the coordinate transformation that we need to perform induces total derivative terms quadratic in the gauge transformation parameters ξ^i. The quadratic terms in the symplectic form can be obtained from the linear ones simply by substituting $\delta g_{mn} \rightarrow \nabla_{(m}\xi_{n)}$. The above argument holds since a pure gauge satisfies the linearized equations of motion.

We observe that H^{tmn} go to zero fast enough close to the curve not to give any contribution from the three form term in the action. The only terms that contribute coming from the gravitational total derivative [33] are

$$\Delta J^t = -\sqrt{-g}\,\nabla_n\left[c^n\left(\delta g_{m,n}, \xi^l\right) + \frac{1}{2}c^n\left(\nabla_{(m}\xi_{n)}, \xi^l\right)\right]$$

$$c^n\left(\delta g_{m,n}, \xi^l\right) = \left(\nabla_m\delta g^{mn} + \nabla^n\delta\ln g\right)\wedge\xi^t + \nabla^n\delta g^{mt}\wedge\xi_m$$
$$+\nabla_m\xi^t\wedge\delta g^{mn} + \frac{1}{2}\nabla^n\xi^t\wedge\delta\ln g - (t \leftrightarrow n)$$

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As we see in the appendix after integrating the above expression and taking the appropriate limit we have

$$\Delta J^t = 2\pi \int_0^L du \frac{1}{\left|\dot{F}\right|^2} \left(\dot{F}_2 \delta \dot{F}_1 - \dot{F}_1 \delta \dot{F}_2 \right) \wedge \left(\dot{F}_2 \delta F_1 - \dot{F}_1 \delta F_2 \right)$$

After we add the contribution that comes from the total derivative term we have the final form [2]

$$J^t = 2\pi \int_0^L du \delta \dot{F}_i \wedge \delta F_i.$$

4.2   Non-linear dynamics

In this section we would like to consider the full dynamics of the system. For simplicity we will concentrate on two of the degrees of freedom. The analysis in general will be analogous. One can write down an action consisting of the symplectic form (derived in the previous section) plus the angular momentum of the NS sector.

$$S = \int dt\, du \left[\dot{F}_1 F_1' + \dot{F}_2 F_2' + F_1 F_2' - F_2 F_1' - F_1'^2 - F_2'^2 \right]$$

Our goal is to relate this dynamical system to the collective dynamics given in sect. 2. We will demonstrate agreemnt by a change of variables

$$
\begin{aligned}
F_1(t, u) &= r(t, u) \cos(\phi(t, u)) \\
F_2(t, u) &= r(t, u) \sin(\phi(t, u)).
\end{aligned}
$$

The action now takes the form

$$S = \int dt\, du \left[\dot{r} r' + \dot{\phi} \phi' r^2 + \phi' r^2 - r'^2 - r^2 \phi'^2 \right]$$

while the non-trivial constrain now reads

$$\int_0^{2\pi} \left[r^2 \phi'^2 + r'^2 \right] du = 2\pi Q_1 Q_5.$$

We may shift the field ϕ by a classical background

$$
\begin{aligned}
\phi &\rightarrow u + \tilde{\phi} \\
r &\rightarrow r_0 + \tilde{r}
\end{aligned}
$$

---

[2]The expected form for this symplectic form was described in Rychkov,talk at Strings 2005, also [47]

and also split the the field into a zero mode r_0 and its fluctuating part which we denote by $\tilde{r}$. After doing so one can show, by non-linear field redefinitions, that up to cubic order in fluctuations the system is described by the action

$$S = \int dtdu \left[\hat{D}\dot{r} \int^u du' \hat{D}r + \dot{z} \int^u du' z + \hat{D}r\hat{D}r - z^2 \right] - 2\pi \int dt\, r_0^2$$

where

$$\hat{D} = i\partial_u + 1$$
$$z = r_0\phi' + \tilde{r}.$$

In order to make contact with the fermion droplet in the action angle representation we redefine

$$\rho^2 = \hat{D}r.$$

As we have seen in the perturbative analysis the field ρ^2 is directly related to the gravity chiral primary field σ and z is related to the field σ^{10}. The action now reads,

$$S = \int dtdu \left[\dot{\rho}^2 \int^u du' \rho^2 + \dot{z} \int^u du' z + \rho^4 - z^2 \right] - 2\pi \int dt\, \rho_0^4.$$

In order to make contact with the non-linear chiral boson we perform the field dependent coordinate transformation

$$x = \rho\cos u$$
$$y_\pm = \rho\sin u$$

and the action now reads

$$
\begin{aligned}
S = \quad & \int dt\, dx \left[4\dot{y}_+ \int^x dx' y_+ - 4\dot{y}_- \int^x dx' y_- - \frac{4}{3}\left(y_+^3 - y_-^3\right) - 4x^2\left(y_+ - y_-\right) \right] \\
+ \quad & \int dt\, dx\, \frac{x\partial_x y_+ - y_+}{y_+^2 + x^2} \left[\dot{z}_+ - \frac{x\dot{y}_+}{x\partial_x y_+ - y_+}\partial_x z_+ \right] \int^x dx' \frac{x\partial_x y_+ - y_+}{y_+^2 + x^2} z_+ \\
- \quad & \int dt\, dx\, \frac{x\partial_x y_- - y_-}{y_-^2 + x^2} \left[\dot{z}_- - \frac{x\dot{y}_-}{x\partial_x y_- - y_-}\partial_x z_- \right] \int^x dx' \frac{x\partial_x y_- - y_-}{y_-^2 + x^2} z_- \\
- \quad & \int dt\, dx \left[\frac{y_+ - x\partial_x y_+}{y_+^2 + x^2}z_+^2 - \frac{y_- - x\partial_x y_-}{y_-^2 + x^2}z_-^2 \right] - 2\pi \int dt\, \rho_0^4.
\end{aligned}
$$

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At this point we shift by the background

$$y_{\pm}^0 = \pm\sqrt{\rho_0^2 - x^2} = \pm\phi^0$$
$$\rho_0^2 \approx \sqrt{Q_1 Q_5}$$

which corresponds to the $AdS_3 \times S^3$ background in the decoupling limit of the $D1 - D5$ system. We also need to perfom the shifts

$$\pm z_{\pm} \rightarrow \frac{1}{2\rho_0^2}\phi^0\partial_x\left[\phi^0\partial_x(xy_{\pm})\int^x \frac{dx'}{\phi^0}z_{\pm}\right] + \frac{1}{2\rho_0^2}\phi^0\partial_x(xy_{\pm})z_{\pm}$$

$$\pm y_{\pm} \rightarrow -\frac{1}{8\rho_0^2}\partial_x\left[x\partial_x z_{\pm}\int^x \frac{dx'}{\phi^0}z_{\pm}\right]$$

so that we will have the Poisson brackets

$$\{y_{\pm}(x), y_{\pm}(x')\} = \pm 4\partial_x\delta(x - x')$$
$$\{z_{\pm}(x), z_{\pm}(x')\} = \pm\phi^0(x)\partial_x\left[\phi^0(x)\delta(x - x')\right].$$

The resulting Hamiltonian is given by

$$
\begin{aligned}
H = & \int dt\,dx\left[4\phi^0\left(y_+^2 + y_-^2\right) + \frac{1}{q^0}\left(z_+^2 + z_-^2\right)\right] + \frac{4}{3}\int dt\,dx\left(y_+^3 - y_-^3\right) \\
+ & \frac{1}{\rho_0^2}\int dt\,dx\left[z_+\partial_x\left[\partial_x(xy_+)\int^x \frac{dx'}{\phi^0}z_+\right] - \phi^0 y_+\partial_x\left[x\partial_x z_+\int^x \frac{dx'}{\phi^0}z_+\right]\right] \\
- & \frac{1}{\rho_0^2}\int dt\,dx\left[z_-\partial_x\left[\partial_x(xy_-)\int^x \frac{dx'}{\phi^0}z_-\right] - \phi^0 y_-\partial_x\left[x\partial_x z_-\int^x \frac{dx'}{\phi^0}z_-\right]\right].
\end{aligned}
$$

The structure of the above Hamiltonian is of the form

$$H = H(y) + H_{int}$$

where $HH(y)$ is the cubic collective Hamiltonian of the free fermions and

$$H_{int} = y_+ T(z_+) + y_- T(z_-)$$

has the form of the interacting Hamiltonian given in the first section.Let us recapitulate what was done in this subsection. Starting with the linear F-system we performed a change of variables relating the gravitational $1/2$ BPS configuration to the interacting picture of S_N orbifold collective field theory. The change of variables was deduced from the linearized analysis (giving the physical degrees of freedom in sect.3) and the change from

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angular to cartesian coordinates known in the case of a pure fermion droplet. The existence of such a change of variables relating the linear with the nonlinear version of our theory is associated with the integrable property of the system. In the CFT or matrix model version this integrable feature was already identified in ref.16,it can also be interpreted as coming from the integrability of the matrix-vector model described in Sect.2.

5   Conclusions

In this paper we have considered in some detail the hamiltonian descriptions of BPS states in AdS3 xS3 Sugra . Elaborating on the construction of [16] we have given a summary of the collective field description which was associated with the correlators of the SN orbifold conformal field theory. We have identified a simple set of elementary coordinates responsible for the dynamics of the BPS sector of the theory. The system consists of a set of eigenvalu coordinates plus the fermionic zero modes of the compactifying manifold X .As such it extends the fermion droplet of the D3 brane theory ,here one has a fermion droplet interacting with boundary degrees of freedom. Direct calculations in AdS3xS3xT4 Sugra were performed to exibit the analogous structure from supergravity. To this end the evaluation of the symplectic form for the general BPS configuratin in Sugra is given. After an identification of physical degrees of freedom a comparison with the collective field picture is accomplished. This involved a transformation from the action-angle variables desribing the general D1D5 2-charge configuration to a physical set of variables. Further studies of this system are clearly of interest.This involves a correspondence with the 'supertube' picture and the question of its quantum mechanical description. Extension to more general 3-charge configurations are of further major interest as they offer a promise for studies of black hole configurations in this approach.

Acknowledgments

A   Appendix A

A.1 $AdS_3 \times S^3$

The way that we may see $AdS_3 \times S^3$ is by setting

$$
\begin{aligned}
F_1 &= \sqrt{Q_1 Q_5} \cos\left(\frac{u}{Q_5}\right), \; F_2 = \sqrt{Q_1 Q_5} \sin\left(\frac{u}{Q_5}\right) \\
x_1 &= \sqrt{Q_1 Q_5} \sqrt{r^2 + 1} \sin\theta \cos\phi \\
x_2 &= \sqrt{Q_1 Q_5} \sqrt{r^2 + 1} \sin\theta \sin\phi \\
x_3 &= \sqrt{Q_1 Q_5} \, r \cos\theta \cos\psi \\
x_3 &= \sqrt{Q_1 Q_5} \, r \cos\theta \sin\psi.
\end{aligned}
$$

Plugging the above to the solution we find the harmonic functions

$$
f^0_{1,5} = \frac{Q^{-1}_{5,1}}{r^2 + \cos^2\theta}
$$

$$
A^0_\phi = \frac{\sin^2\theta}{r^2 + \cos^2\theta}
$$

$$
B^0_\psi = \frac{\cos^2\theta}{r^2 + \cos^2\theta}
$$

which yield the six dimensional Einstein frame metric

$$
\frac{ds^2_6}{\sqrt{Q_1 Q_5}} = \left(r^2 + \cos^2\theta\right)\left[-\left(dt - \frac{\sin^2\theta d\phi}{r^2 + \cos^2\theta}\right)^2 + \left(d\chi + \frac{\cos^2\theta d\psi}{r^2 + \cos^2\theta}\right)^2\right]
$$

$$
+\frac{1}{r^2 + \cos^2\theta}\left[\left(r^2 + \cos^2\theta\right)\left(\frac{dr^2}{r^2 + 1} + d\theta^2\right) + r^2\cos^2\theta d\phi^2 + \left(r^2 + 1\right)\sin^2\theta d\phi^2\right].
$$

After perfoming the change of coordinates

$$
\tilde\phi = \phi - t, \; \tilde\psi = \psi + y
$$

we find the $AdS_3 \times S^3$ metric in global coordinates

$$
\frac{ds^2_6}{\sqrt{Q_1 Q_5}} = -\left(r^2 + 1\right)dt^2 + \frac{dr^2}{r^2 + 1} + r^2 dy^2 + d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2.
$$

A.2   From $F_1(u), F_2(u)$ to $\rho(\phi), u(\phi)$

In the next section we correspond modes of the curve to modes of the two chiral primaries σ_n and s^5_n where n is a quantum number that has to do

with the isometry of the background metric related to the killing vector ∂_ϕ. As we can see from the coordinate transformation

$$x_1 = \sqrt{Q_1 Q_5} \sqrt{r^2 + 1} \sin\theta \cos\phi$$
$$x_2 = \sqrt{Q_1 Q_5} \sqrt{r^2 + 1} \sin\theta \sin\phi$$

the angle ϕ is related to the angle $\phi = \arctan\left(\frac{F_2(u)}{F_1(u)}\right)$. We now perform the change of variables

$$\rho^2(\phi) = F_1^2(u(\phi)) + F_1^2(u(\phi))$$
$$\phi = \arctan\left(\frac{F_2(u)}{F_1(u)}\right)$$

After inverting the last equation one is able to express u in terms of ϕ. In terms of the new variables the functions calculated from the curve $\rho(\phi), u(\phi)$ read

$$f_5 = 1 + \frac{1}{2\pi}\int_0^{2\pi} d\tilde\phi \frac{u_{,\tilde\phi}}{d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2} \approx \frac{1}{2\pi}\int_0^{2\pi} d\tilde\phi \frac{u_{,\tilde\phi}}{d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2}$$

$$f_1 = 1 + \frac{1}{2\pi}\int_0^{2\pi} \frac{d\tilde\phi}{u_{,\tilde\phi}} \frac{\rho_{\tilde\phi}^2 + \rho^2}{d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2} \approx \frac{1}{2\pi}\int_0^{2\pi} \frac{d\tilde\phi}{u_{,\tilde\phi}} \frac{\rho_{\tilde\phi}^2 + \rho^2}{d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2}$$

$$A_\phi = \frac{\sqrt{Q_1 Q_5}\sqrt{r^2+1}\sin\theta}{2\pi}\int_0^{2\pi} d\tilde\phi \frac{\rho\cos\left(\phi-\tilde\phi\right) + \sin\left(\tilde\phi - \phi\right)\partial_{\tilde\phi}\rho}{d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2}$$

$$A_r = \frac{\sqrt{Q_1 Q_5}\,r\sin\theta}{2\pi\sqrt{r^2+1}}\int_0^{2\pi} d\tilde\phi \frac{\partial_{\tilde\phi}\rho\cos\left(\phi-\tilde\phi\right) + \rho\sin\left(\phi-\tilde\phi\right)}{d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2}$$

$$A_\theta = \frac{\sqrt{Q_1 Q_5}\sqrt{r^2+1}\cos\theta}{2\pi}\int_0^{2\pi} d\tilde\phi \frac{\partial_{\tilde\phi}\rho\cos\left(\phi-\tilde\phi\right) + \rho\sin\left(\phi-\tilde\phi\right)}{d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2}$$

$$B_\psi = -A_\phi - \frac{1}{2\pi}\int_0^{2\pi} d\tilde\phi \frac{\rho^2}{d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2}$$

where

$$d\left(\rho,\theta,\phi;\rho\left(\tilde\phi\right),\tilde\phi\right)^2 = Q_1 Q_5\left(r^2 + \sin^2\theta\right) + \rho^2\left(\tilde\phi\right) - 2\sqrt{Q_1 Q_5}\sqrt{r^2+1}\sin\theta\rho\left(\tilde\phi\right)\cos\left(\phi-\tilde\phi\right)$$

In terms of the new variables the vacuum is given by

$$\rho_0 = \sqrt{Q_1 Q_5}, \ u_0 = Q_5 \phi$$

B   Appendix B

B.1   Details of the evaluation of the symplectic form

The non zero components of the inverse of the metric are given by

$$g^{tt} = \frac{-f_1 f_5 + A^2}{\sqrt{f_1 f_5}}, \ g^{yy} = \frac{f_1 f_5 + B^2}{\sqrt{f_1 f_5}},$$

$$g^{x^i x^j} = \delta^{ij} \frac{1}{\sqrt{f_1 f_5}}, \ g^{yx^i} = -\frac{B^i}{\sqrt{f_1 f_5}}, \ g^{tx_i} = \frac{A_i}{\sqrt{f_1 f_5}}$$

and the determinant is given by

$$g = -f_1 f_5.$$

It is useful to see the components of the matrix

$$j^{mn} = \sqrt{-g} g^{mn}$$

which are given by

$$j^{tt} = -f_1 f_5 + A^2, \ j^{yy} = f_1 f_5 + B^2$$
$$j^{x^i x^j} = \delta^{ij}, \ j^{yx^i} = -B^i, \ j^{tx_i} = A_i$$

The nonzero Christoffel symbols that we will need are given by

$$\Gamma^t_{tt} = -\frac{A_i \partial_i \sqrt{w}}{2\sqrt{w}^3}$$

$$\Gamma^t_{yy} = \frac{A_i \partial_i \sqrt{w}}{2\sqrt{w}^3}$$

$$\Gamma^t_{x^i t} = \frac{1}{2\sqrt{w}^3} \left[\sqrt{w} A_j \partial_i A_j - \sqrt{w} A_j \partial_j A_i + A_i A_j \partial_j \sqrt{w} - \sqrt{w}^2 \partial_i \sqrt{w} \right]$$

$$\Gamma^t_{x^i y} = \frac{1}{2\sqrt{w}^3} \left[\sqrt{w} A_j \partial_i B_j - \sqrt{w} A_j \partial_j B_i + B_i A_j \partial_j \sqrt{w} \right]$$

$$\Gamma^n_{n x^i} = \frac{\partial_i \sqrt{w}}{\sqrt{w}}$$

Where we have set $w = f_1 f_5$. The gravitational contribution from the bulk gives

$$
\begin{aligned}
J_G^t = \quad & -\delta\left(\frac{A_i \partial_i \sqrt{w}}{\sqrt{w}^3}\right) \wedge \delta(w) - \delta\left(\frac{A_i \partial_i \sqrt{w}}{2\sqrt{w}^3}\right) \wedge \delta A^2 + \delta\left(\frac{A_i \partial_i \sqrt{w}}{2\sqrt{w}^3}\right) \wedge \delta B^2 \\
& -\delta\left[\frac{1}{\sqrt{w}^3}\left[\sqrt{w} A_j \partial_i A_j - \sqrt{w} A_j \partial_j A_i + A_i A_j \partial_j \sqrt{w} - \sqrt{w}^2 \partial_i \sqrt{w}\right]\right] \wedge \delta A_i \\
& +\delta\left[\frac{1}{\sqrt{w}^3}\left[\sqrt{w} A_j \partial_i B_j - \sqrt{w} A_j \partial_j B_i + B_i A_j \partial_j \sqrt{w}\right]\right] \wedge \delta B_i \\
& +\delta\left(\frac{\partial_i \sqrt{w}}{\sqrt{w}}\right) \wedge \delta A_i
\end{aligned}
$$

After a little simplification we have the form

$$
\begin{aligned}
J_G^t = \quad & -\delta\left(\frac{A_i \partial_i \sqrt{w}}{\sqrt{w}^3}\right) \wedge \delta(w) \\
& -\delta\left[\frac{1}{\sqrt{w}^3}\left[\sqrt{w} A_j \partial_i A_j - g A_j \partial_j A_i\right]\right] \wedge \delta A_i \\
& +\delta\left[\frac{1}{\sqrt{w}^3}\left[\sqrt{w} A_j \partial_i B_j - \sqrt{w} A_j \partial_j B_i\right]\right] \wedge \delta B_i \\
& +2\delta\left(\frac{\partial_i \sqrt{w}}{\sqrt{w}}\right) \wedge \delta A_i
\end{aligned}
$$

The final result for the case of the gauge field part reads

$$
\begin{aligned}
J_H^t = \quad & -\frac{1}{2}\delta\left[2\frac{1}{f_5}\partial_{[i} B_{j]} + \frac{3\partial_{[i} \mathcal{C}_{ja]}}{f_5^2} A^a\right] \wedge \delta\left[\mathcal{C}_{ij} + \frac{2}{f_1} B_{[i} A_{j]}\right] \\
& -\delta\left[-f_1^2 \partial_i \frac{1}{f_1} + \frac{1}{f_5}\left[B^b \partial_i B_b - B^b \partial_b B_i - A^a \partial_i A_a + A^a \partial_a A_i\right] - \frac{1}{f_5^2}3\partial_{[a}\mathcal{C}_{bi]}A^a B^b\right] \wedge \delta\left[\frac{A_i}{f_1}\right]
\end{aligned}
$$

For the contribution of the scalar we simply have

$$
J_a^t = -\frac{1}{2}\delta\left(A^i \partial_i \ln\frac{f_1}{f_5}\right) \wedge \delta \ln\frac{f_1}{f_5}.
$$

Putting everything together we have that the bulk gives

$$
\begin{aligned}
J^t_{bulk} = \int d^4x \quad & \left\{ -\frac{1}{2}\delta\left(A^i\partial_i\ln\frac{f_1}{f_5}\right)\wedge\delta\ln\frac{f_1}{f_5} + \delta\left[\frac{1}{f_5^2}3\partial_{[a}\mathcal{C}_{bi]}A^aB^b\right]\wedge\delta\left[\frac{A_i}{f_1}\right] \right. \\
& -\frac{1}{2}\delta\left[2\frac{1}{f_5}\partial_{[i}B_{j]} + \frac{3\partial_{[i}\mathcal{C}_{ja]}}{f_5^2}A^a\right]\wedge\delta\left[\mathcal{C}_{ij} + \frac{2}{f_1}B_{[i}A_{j]}\right] \\
& -\delta\left[-f_1^2\partial_i\frac{1}{f_1} + \frac{1}{f_5}\left[B^b\partial_iB_b - A^a\partial_iA_a + A^a\partial_iA_a\right]\right]\wedge\delta\left[\frac{A_i}{f_1}\right] \\
& -\delta\left(\frac{A_i\partial_i\sqrt{w}}{\sqrt{w}^3}\right)\wedge\delta\left(w\right) + 2\delta\left(\frac{\partial_i\sqrt{w}}{\sqrt{w}}\right)\wedge\delta A_i \\
& -\delta\left[\frac{1}{\sqrt{w}^3}\left[\sqrt{w}A_j\partial_iA_j - gA_j\partial_jA_i\right]\right]\wedge\delta A_i \\
& \left. +\delta\left[\frac{1}{\sqrt{w}^3}\left[\sqrt{w}A_j\partial_iB_j - \sqrt{w}A_j\partial_jB_i\right]\right]\wedge\delta B_i \right\}
\end{aligned}
$$

After many cancelations because of the non-commutative nature

$$\delta A\wedge\delta B = -\delta B\wedge\delta A$$

and the linear equations

$$
\begin{aligned}
dA &= -*_4\,dB \\
d\mathcal{C} &= -*\,df_5
\end{aligned}
$$

we can bring the integrand to a total divergence form

$$J^t_{bulk} = -\int d^4x\,\partial_d\left\{\frac{1}{2}\epsilon_{ijad}\delta\left[\frac{A^a}{f_5}\right]\wedge\delta\left(\mathcal{C}_{ij}\right)\right\}.$$

B.2 The total derivative terms

It is interesting to see how a one dimensional integral is obtained from the above surface integrals. At this point we would like to introduce a surface of topology $S^1\times S^2$ of radius surrounding the ring. Using Gauss' law we are able to reduce the calculation of the total derivative to a surface integral on the above mentioned surface. The embedding in the four dimensional space spanned by $x^i, i = 1\ldots 4$ is

$$
\begin{aligned}
&\vec{x}\left(u,\theta,\phi\right) = x_\perp\left(\ \sin\theta\frac{\dot{F}_2}{|\dot{F}|} + F_1,\quad -\sin\theta\frac{\dot{F}_1}{|\dot{F}|} + F_2,\quad \cos\theta\cos\phi,\quad \cos\theta\sin\phi\ \right) \\
&u\,\epsilon\,[0, 2\pi Q_5]\,,\ \theta\,\epsilon\,[0,\pi]\,,\ \phi\,\epsilon\,[0, 2\pi)
\end{aligned}
$$

From the above we see that the measure of integration on the tube is given by

$$dS = x_\perp^2 \left| \dot{F} \right| \sin\theta \, du \, d\theta \, d\phi$$

and the normal unit vector is

$$\vec{n}\left(u, \theta, \phi\right) = \left(\ \sin\theta \frac{\dot{F}_2}{|\dot{F}|}, \ \ -\sin\theta \frac{\dot{F}_1}{|\dot{F}|}, \ \ \cos\theta\cos\phi, \ \ \cos\theta\sin\phi \ \right).$$

At the end of the calculation we want to consider the limit $x_\perp \to 0^+$ which leads us to keeping the terms of order $O\left(x_\perp^{-2}\right)$. We can indeed show that this is the leading behavior of the integrand.

From the expressions that were previously given we have the variations

$$
\begin{aligned}
\delta f_5 &= \frac{2Q_5}{L} \int_0^L \frac{du}{|x-F|^4} \left(x_i - F_i\right) \delta F_i \\[2mm]
\delta f_1 &= \frac{Q_5}{L} \int_0^L \frac{\delta|\dot{F}|^2 du}{|x-F|^2} + \frac{2Q_5}{L} \int_0^L \frac{|\dot{F}|^2 du}{|x-F|^4} \left(x_i - F_i\right) \delta F_i \\[2mm]
\delta A_j &= -\frac{Q_5}{L} \int_0^L du \frac{\delta F_{j,u}}{|x-F|^2} - \frac{2Q_5}{L} \int_0^L du \frac{F_{j,u}}{|x-F|^4} \left(x_i - F_i\right) \delta F_i
\end{aligned}
$$

We would like to approximate the above expressions in the limit where $\mathbf{x}$ approaches some point on the curve $F\left(u\right)$

$$
\begin{aligned}
\delta f_5 &\approx \frac{1}{2|\dot{F}|x^{\perp 2}} n_i \delta F_i + O\left(x_\perp^{-1}\right) \\[2mm]
\delta f_1 &\approx \frac{|\dot{F}|}{2x^{\perp 2}} n_i \delta F_i + O\left(x_\perp^{-1}\right) \\[2mm]
\delta A_j &\approx -\frac{\dot{F}_j}{2|\dot{F}|x^{\perp 2}} n_i \delta F_i - \frac{\delta\dot{F}_j}{2|\dot{F}|x^\perp} + O\left(x_\perp^0\right).
\end{aligned}
$$

We can also use the asymptotic expansions

$$
\begin{aligned}
f_1 &\approx \frac{|\dot{F}|}{2x^\perp} \\[2mm]
f_5 &\approx \frac{1}{2|\dot{F}|x^\perp} \\[2mm]
A_i &\approx -\frac{\dot{F}_i}{2|\dot{F}|x^\perp}
\end{aligned}
$$

which give the useful identity

$$n_i g^{txi} = -\frac{n_i \dot{F}_i}{2|\dot{F}|} + O(x_\perp) = O(x_\perp)$$

The above expansion enables us to determine the asymptotic behavior of the covariant derivative also. The contribution of the gauge transformation term is given by

$$\int d^4x \partial_d \left\{ \sqrt{-g} g^{dm} g_{rq} \nabla_m \delta g^{rt} \wedge \xi^q - \sqrt{-g} \nabla_m \xi^d \wedge \delta g^{mt} \right\}$$

$$= 2\pi \int_0^L du \frac{1}{\left|\dot{F}\right|^2} \left(\dot{F}_2 \delta \dot{F}_1 - \dot{F}_1 \delta \dot{F}_2 \right) \wedge \left(\dot{F}_2 \delta F_1 - \dot{F}_1 \delta F_2 \right)$$

The calculation of the term coming from the bulk is a little more involved. The equation that needs to be solved for C_{ij} is a monopole type equation. As usually we will need to solve the equation on patches. In order to make the calculation easier we will define the fourth direction to be the direction defined by the tangent on the ring at some point $F(u)$. The gauge that we fix is

$$C_{4i} = 0.$$

The equations now take the form

$$\partial_4 C_{\alpha\beta} = \epsilon_{\alpha\beta\gamma} \partial_\gamma f_5$$
$$\frac{1}{2} \epsilon_{\alpha\beta\gamma} \partial_\alpha C_{\beta\gamma} = \partial_4 f_5$$

Where $\alpha, \beta, \gamma = 1, 2, 3$. For the variation we have

$$\partial_4 \delta C_{\alpha\beta} = \epsilon_{\alpha\beta\gamma} \partial_\gamma \delta f_5$$
$$\frac{1}{2} \epsilon_{\alpha\beta\gamma} \partial_\alpha \delta C_{\beta\gamma} = \partial_4 \delta f_5.$$

Solving the equations close to the ring yields

$$\delta C_{\alpha\beta} = \epsilon_{\alpha\beta\gamma} \frac{n_\gamma \delta F}{2x_\perp}$$

where

$$\delta F = \frac{\dot{F}_i \delta F_i}{\left|\dot{F}\right|}$$

27

is the variation parallel to the ring. The integral that we need to perform now reads

$$\int d^4x\, \partial_d \left[\frac{1}{2}\epsilon_{ijad}\delta\left[\frac{A^a}{f_5}\right] \wedge \delta\left(C_{ij}\right)\right] = -2\pi \int_0^L du \frac{1}{\left|\dot{F}\right|^2}\left(\dot{F}_1\delta\dot{F}_1 + \dot{F}_2\delta\dot{F}_2\right) \wedge \left(\dot{F}_1\delta F_1 + \dot{F}_2\delta F_2\right)$$

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